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सत्रीय कार्य / Assignment Work – 2019-20

एम.एस.सी. (पूर्व) गणित (M.Sc. Mathematics Pre)

Max Marks – 30 **Min Marks-12**

निर्देश : सत्रीय कार्य के प्रत्येक विषय में कुल 30 अंक हैं। सभी प्रश्नों के अंक समान होंगे। सभी प्रश्न हल कीजिए। (Assignment Work of each paper carries 30 Marks. All questions carry equal marks. Attempt all questions.)

Paper-I : Advanced Abstract Algebra

1. Prove that a non-commutative group must have at least six elements. Give an example of a non—commutative group of order 6.
2. Which of the following mappings are group homomorphisms ? In case they are group homomorphisms find the kernel.
   (i) $f: R-\{0\} \to R-\{0\}$ under multiplication defined by
   $f(x) = |x|,$
   (ii) $f: Z \to R$ under addition defined by $f(x) = x, and$
3. Show that if N is a proper normal subgroup of a group G Which has a composition series, than there exist a composition series containing N.
4. A units R-module M is cyclic if and only if M $\cong R/I$ for some left ideal I in R.
5. If K is an extension of F, prove that the set of elements in K Which are separable over F forms a subfield of K

Paper-II : Real Analysis

1. Show that if f is integrable on [a, b], then

$$\underline{\int_a^b} f(x) = \int_a^b f(x)\, dx .$$

2. Determine convergence or divergence
   (a) $\sum \frac{\sqrt{n^2-1}}{\sqrt{n^5+1}}$ (b) $\sum \frac{1}{n^2\left[1+\frac{1}{2}\sin(n\pi/4)\right]}$
3. Give a geometrical interpretation of the directional derivative $\partial f(x_0, y_0)/\partial \Phi$ of a function of two variables.
4. Suppose that F and G are transformations from R $\to$ R have the same domain D. Show that if F and G are continuous at $X_0 \in D$, then so are F+G and F – G.
5. Prove : If f is continuous on [a,b] × [c,d], then the function

$$F(y) = \int_a^b f(x,y)\, dx$$

is continuous on [c, d].

Paper-III : Topology

1. (a) Prove that $|d(x, A) - d(y, A)| \leq d(x, y)$

   (b) Let A be a subset of a metric space. There prove that

$$\bar{A} = A \cup D\ (A)$$

2. (a) Show that $\pi, (s^1, 1) \cong \mathbb{Z}$

   (b) State and prove Van Kampen theorem.

3. Let $i : A \ \to X$ be a cofibration, where A is contractible space. Prove that the quotient map $X \to X/A$ is a homotopy equivalence.

4. Show that, if $n \geq 2, then\ \pi_n\ (XVY)$ is isomorphic to

$$\pi_n\ (X) \oplus \pi_n\ (Y) \oplus \pi_{n+1}\ (X \times Y,\ X\ V\ Y)\ .$$

5. Show that $Y\ U_f\ X$ is a CW complex with Y as a subcomplex and X/A as a quotient complex. Formulate and prove a formula relating the Euler characteristics $x(A), x(X), and\ x\big(Y \cup_f\ X\big) when\ X\ Y\ are\ finite$.

Paper-IV : Complex Analysis

1. Show that the function

   $u = \sin x \cosh y + 2 \cos x \sinh y + \ x^2\ -\ y^2\ +\ 4xy$

2. Find the bilinear transformation which maps

   $z_1\ = 1, z_2\ = 1\ \ and\ z_3\ \ = \ \infty\ into\ the\ point\ w_1$ = -I, $w_2\ \ =\ -1\ \ and\ \ w_3\ = i.$

3. Examine the behavior of the following power series on the circle of convergence :

   (i) $\sum_1^\infty \frac{Z^n}{n}$ (ii) $\sum_2^\infty \frac{Z^{4n}}{4n+1}$

4. $If\ the\ function\ f\ (z) in\ analytic\ and\ one\ valued\ in\ \left|z - a\right| < R,$

   $prove\ that\ when\ 0 < r < R,$

$$f(a) = \frac{1}{\pi r} \int_0^{2\pi} P\ (\theta)\ e^{-i\theta}\ \ d\theta$$

   $where\ p\ (\theta) is\ the\ real\ part\ of\ f\ \left(a + re^{i\theta}\right).$

5. Show that the product

$$\left\{\left(1 - \frac{z}{1}\right) e^z\right\}\ \left\{\left(1 + \frac{z}{1}\right) e^{-z}\right\}\left\{\left(1 - \frac{z}{2}\right) e^{z/2}\right\}\left\{\left(1 + \frac{z}{2}\right) e^{-z/2}\right\}\ \dots ....$$

   $converges\ absolutely\ and\ uniformly\ ub\ any\ closed\ D\ which\ does\ not\ contain\ any$

   $of\ the\ point\ \ \pm 1, \pm 2, \dots \dots ....$

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